*University of Toronto*

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# STAB57: an Introduction to Statistics

Week 9 Assignment

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-website

-textbook

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This week's list of problems is based on the material from:
Chapter 6, §3, Chapter 7, §1, §2
You are expected to work on this list of problems prior to the upcoming tutorial.
Problems have the following tags:
difficult, Book exercise, extra exercise

**Terminology and Concepts to learn:**

- Bayesian model: prior, prior predictive, posterior distribution
- Bayesian inference: estimation through mean and mode for Bernoulli and location normal model
- conjugate priors
- Bayesian inference: credible interval
- Bayesian inference: p-value and Bayesian factor

## Problem 1

Assume that a probability distribution $P$ has a mean $\mu$ and mode $m$. Assume that the distribution is symmetric in the sense that the density function satisfies $f(x_0 + x) = f(x_0 - x)$ for some $x_0$ Show that $\mu = m = x_0$

## Problem 2

Practice your skills on Bayesian estimation by doing problems: 7.2.1, 7.2.3, 7.2.5, 7.2.6, 7.2.10 and 7.2.11

## Problem 3

Consider a Bayesian model with a uniform prior on $\Theta$ and a statistical model $\Theta \Longrightarrow S$ where $f_\theta \sim$ Geometric($\theta$). Compute the posterior. Can you find a conjugate prior distribution in this case?

## Problem 4

Similarly, assume $f_\theta \sim$ exponential($\theta$) in this case. Find the posterior density and determine a conjugate prior.

## Problem 5

We alluded to the cat that another way to test hypotheses is through Bayesian factors, which uses odds as opposed to probabilities. ie $O(A) = P(A)/(1 - P(A))$. Another application of odds is the function $\phi : [0, 1[ \longrightarrow \mathbb{R} : p \mapsto \ln(\frac{p}{1-p})$. Show that this function is ono-to-one and onto (alternatively, show that this function has an inverse).[1]

[1] this function will play a key role in the theory of regression