# *University of Toronto*

## *Department of Computer & Mathematical Sciences*

# MATC32: Graph theory and applications

Things you should know for the final

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*this document contains a detailed list of the things you should know in order to prepare for the final exam*
*the exam takes place on Monday, December 18th at 7pm in IC 220*

**Planarity of Graphs Ch. 6**

- You should know what a graph drawing in $\mathbb{R}^n$ is: this includes polygonal curves and crossings (§6.1)
- You should know Euler's formula (6.1.21) and understand its proof (using edge contraction and the Jordan curve theorem 6.1.6)
- You should be able to apply this formula to obtain important inequalities (such as $E \leq 3V - 6$ and $E \leq 2V - 4$ 6.2.23).
- You should know Kuratowski's theorem and understand why the conditions are necessary (6.2.1-6.2.2)
- you should know what the dual of a planar graph is and how to use it in certain applications such as 6.1.13

**Platonic solids: 6.1.28**

- You should know what a platonic solid is and which 5 solids are platonic.
- You should understand the different steps in using planarity of graphs to classifying these solids

**the Chromatic number and Brooks' Theorem §5.1**

- You should understand the relation between graph colorings and partitions
- You should know what the chromatic number of a graph is and how to compute it in certain examples
- you shoud know the lower bounds for the chromatic number 5.1.7
- You should be able to execute the greedy coloring algorithm 5.1.12 and understand the upper bound 5.1.13 (in particular how succesor in the algorithm relate to a proper coloring)
- You should know Brooks theorem 5.1.22 and understand its proof in certain cases (in particular how the greedy algorithm gets used)

**3,4,5-6-Colorability: online notes**

- You should know that any planar graph is 5-colorable
- You should understand the proof of 6-colorability (in particular how the two statements imply the result and can be generalized)
- You should understand the proof of 5-colorability and how we manipulated colorings in certain cases

**the Chromatic polynomial (§5.3)**

- You should know what the chromatic polynomial is,and why the function is polynomial in particular
- You should know the contraction-deletion lemma (5.3.6)
- You should be able to apply the lemma to compute the chromatic polynomial of certain graphs such as cycles or trees

**3 Algorithms (§2.3)**

- You should know and be able to execute the two major algorithms we introduced to find a spanning tree: Kruskal's (2.3.1) and Prim's algorithm (ex. 2.3.10)
- You should know and be able to execute Dijkstra's algorithm (2.3.5) to find a shortest path. In particular know how to establish a Dijkstra table for a weighted graph.

**Eulerian graphs ( §1.2)**

- You should know what Eulerian graphs are
- You should know and understand the equivalence between even graphs, decompositions of cycles and Eulerian graphs (1.2.26 and 1.2.27)
- You should be able to apply this theorem to make conclusions about Eulerian graphs.

**Some general comments**

- If you need to understand *understand* a proof, that does not mean you will be asked to reproduce it. However, some of the techniques used will help you solve other questions.
- You will be asked to prove certain statements yourself. In each case, the argument required will be rather short and direct
- Make sure you have as many examples and counterexamples as possible in order to improve your insight into the material
- If you should know/be able to state a result, make sure you can state it correctly! This will be actively tested
- It is a good idea to pick a certain graph and go through the material completely, checking how each result relates to said graph