University of Toronto

Department of Computer & Mathematical Sciences

MATC32: Graph theory

Assignment Nr 4

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-website

This week's list of problems is based off the material covered in weeks 10 through 12: and is covered
You should work on each individual problem and submit your solutions in the MATC32 dropbox by Friday, Dec. 8th
You will be graded not only on your answers but on how you argue. So make sure you prove your arguments correctly or give counterexamples whenever necessary

Hamiltonian Graph

Recall that $K_{m,n}$ denotes the complete bipartite graph on independent sets with m and n vertices respectively.
Find conditions such that $K_{m,n}$ is Hamiltonian.
Prove your claim.
Show that for any n there always exists a Non-Hamiltonian graph whose minimum vertex degree $\geq \frac{|V|}{2} - 1$

Removing a cycle

Let G be a graph with a cycle $C \subset G$. Let $G \setminus C = G_1 \coprod \ldots \coprod G_n$ be the connected components. Show that

$$G_i \cap \Big(G \setminus C \Big) \neq \varnothing$$

the Petersen graph

Does the Petersen graph have a:

- Euler cycle?
- Euler path?

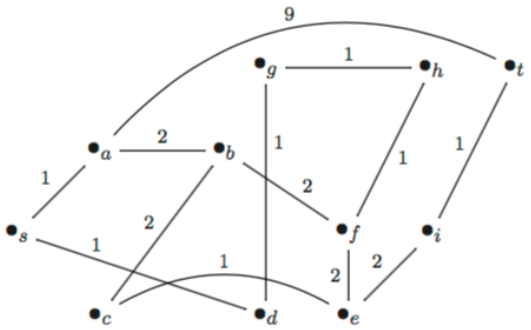
- Hamilton cycle?
- Hamilton path?

Eulerian paths

Show that a graph has an Eulerian path iff it has at most two vertices of odd degree.

Dijkstra

Perform the Dijkstra algorithm to find the shortest path from the vertex s to any other vertex in the following graph:



MST's

Assume the weights in a connected graph are unique. Show that the minimum spanning tree is unique.