University of Toronto

Department of Computer & Mathematical Sciences

MATC32: Graph theory

Assignment Nr 4

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-email me

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This week's list of problems is based off the material covered in weeks 7 through 9:
You should work on each individual problem and submit your solutions in the MATC32
dropbox by Friday, Nov. 17th
You will be graded not only on your answers but on how you argue. So make sure you
prove your arguments correctly or give counterexamples whenever necessary
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3-colourability

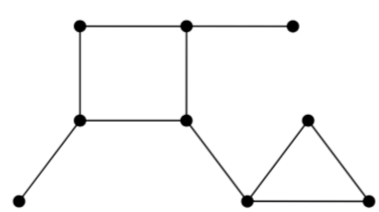
Recall our argument that uses an inequality stemming from Euler's formula to show that a planar graph is 6-colourable. Modify this argument (using a different inequality) to show that a planar graph without triangles is 3-colourable.

the chromatic function

Let $P(G, n)$ denote the amount of different possible proper colourings with n colors on the graph G. Let G be a disjoint union of k edges. Compute $P(G, n)$

Spanning trees

Find all spanning trees for the following graph:



Induction

What is the chromatic number of a cycle? Use induction to formally prove your answer

Computing $\chi(G)$

Consider a graph with vertex set $\{1, \ldots, n\}$ and edge set $\{\{1,2\},\{1,3\}\ldots\{1,n\}\}\cup\{\{2,3\}.\{3,4\}.\{4,5\}\ldots\{n,n-1\}\}$ Draw this graph and compute its chromatic number

the chromatic index

Similar to a vertex covering, we can define an edge covering: $E(G) \longrightarrow C$. An edge coloring is *proper* if no two edges with the same colour are adjacent. The chromatic *index* $\nu(G)$ is the smallest amount of colours required to construct a proper edge coloring.

- For a graph G, the line graph $L(G)$ has vertex set $E(G)$ where two edges are adejacent in $L(G)$ if they share a common endpoint. Find a relation between $\nu(G)$ and $\chi(L(G))$
- use this relation to show that$\Delta \leq \nu(G)$
- Use the relation again together with Brooks' theorem to show that $\nu(G) \leq 2\Delta - 1$