University of Toronto
Department of Computer & Mathematical Sciences
MATC32: Graph theory
Assignment Nr 3

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This week's list of problems is based off the material covered in weeks 5 through 7:
You should work on each individual problem and submit your solutions in the MATC32
dropbox by Friday, Nov.  3rd
You will be graded not only on your answers but on how you argue.  So make sure you
prove your arguments correctly or give counterexamples whenever necessary
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the Euler formula

The Euler formula states that $V - E + F = 2$ for a connected planar graph.
Now, assume that k is a planar graph with k connected components. Use the above formula to show that

$$V - E + F = 1 + k$$

the Petersen graph

- Determine whether the Petersen graph is planar or not.
- Find the length of the shortest cycle (and prove your claim) in the Petersen graph.

A graph coloring

Recall that a n-coloring consists of a coloring of vertices $V \longrightarrow \{1, \ldots, n\}$ such that no two vertices with the same color are adjacent. Find a graph with a 4-coloring but no 3-coloring where each vertex has degree 4.

Planar graphs and degrees

Let n be the smallest integer making the following statement true:

Any planar graph has a vertex of degree of at most n

Find n and prove your statement

Coloring planar graphs

We'll use the above problem to show that a planar graph has a 5-coloring. Let v be a vertex of degree $le5$. And assume by induction that $G \setminus v$ has a 5-coloring

- if v has degree 4, explain how the result follows
- if v has degree 5, find a way to extend the coloring to the whole of G.