University of Toronto

Department of Computer & Mathematical Sciences

MATC32: Graph theory

Assignment Nr 2

taught by Louis de Thanhoffer de Volcsey

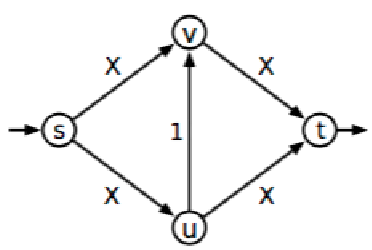
-email me

-website

This week's list of problems is based off the material covered in weeks 3 through 5: i.e. selected topics covered in book sections 3.1 4.2, 4.3 and 4.3.
You should work on each individual problem and submit your solutions in the MATC32 dropbox by Monday, Oct. 16th
You will be graded not only on your answers but on how you argue. So make sure you prove your arguments correctly or give counterexamples whenever necessary

the FF algorithm

Consider the following network:



show that a the Ford-Fulkerton alogrithm can be run in such a way that it only terminates after x steps.

Shuffling cards

Assume you shuffle a deck of cards and split it into 13 piles of 4 cards. Show that you can always make a choice of one card out of each pile wich results in $\{2, 3, 4, \ldots J, Q, K, A\}$

Incidence matrices

Recall that given a vertex set $\{x_1, \dots x_n\}$, the incidence matrix is constructed by letting $M_{i,j}$ be 0 or 1 depending on whether there is an edge between x_i and x_j.
Additionally, recall that a block matrix is a matrix of the following form

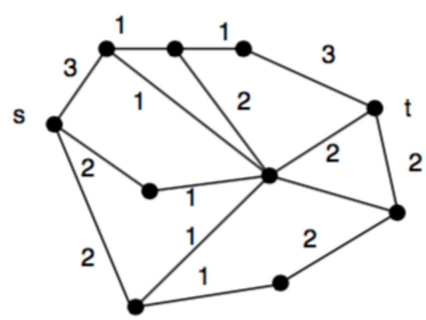
$$C = \left[\begin{array}{c|c} 0 & M \\ \hline N & 0 \end{array}\right]$$

(here 0 simply denotes a zero-matrix Complete and prove the following statement:

$$\text{A graph has an adjacency matrix in block form} \iff \dots$$

the FF algorithm again

Consider the following network:



(if the capacity is not specified, you are free to choose the value).
Find a maximum flow and minimal cut using the Ford-Fulkerton algorithm.

Perfect matching

Let G be a bipartite graph, and let $V(G) = X \coprod Y$ and $|X| = |Y|$. Denote by $d(x)$ the number of edges adjacent to x. Assume there exists a number k such that $d(x) \geq k$ and $d(y) \leq k$ for any $x \in X$, $y \in Y$.
Prove that G has a perfect matching and find an example in the case where $k = 3$.